On Network Management Information For Light-Path Assessment

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***Abstract*--We investigate network management information for lightpath assessment to dynamically set up end-to-end lightpaths across administrative domains. The assessment is based on the partial information which includes aggregated characterization of subnetworks, and local states from wavelength converters. Our focus is on invstigating what performance can be possibly achieved given the partial management information, and to be more specific, whether a small loss in performance can trade off with a large amount of management information, and can thus achieve scalability in management information.**

We cast the light-path assessment as a decision problem, and define the performance as the probability of an erroneous decision. Scalability is defined using the growth rate of the management information that needs to be maintained with respect to the amount of resources within the network (*FHL*). We apply decision theory to show that the optimal performance using one type of partial information (*O*log(*F*)) is the Bayes probability of error. An upper bound of the Bayes error is derived in terms of blocking probability. We evaluate the upper bound using both independent and dependent models of wavelength usage. Our study shows that there exists a "threshold effect": The Bayes error decreases to 0 exponentially with respect to the load when the load is either below or above a threshold value; and is non-negligible when the load is in a small duration around the threshold. This suggests that a small percentage of error decisions can trade off to achieve scalability in management information.

***Index terms*— scalability, management information, lightpath assessment, decision theory, Bayes rule, blocking probability**

I. INTRODUCTION

Dynamically assessing the quality of light-paths is important to many applications in wavelength-routed optical networks such as on-demand light-path provisioning, protection and restoration. As the light-path quality is a complex measure [1], this work considers a simple quality, which is the wavelength availability on a candidate light-path. The assessment then becomes determining availability of wavelengths for supporting an end-to-end call based on given management information.

Complete or partial network management information can be used to assess the wavelength availability on a light-path. Complete states correspond to the detailed information about wavelength usage at each link, i.e. "which wavelengths are used at which parts of a network'' when there is no wavelengths converters in the network. Wavelength converters can reduce state information due to its ability to relax the wavelength continuity constraint. However, it is expected that wavelength converters will remain expensive and are used mostly on the boundaries of sub-networks [2]. Therefore, complete state information involves the detailed wavelength occupancy within a subnet. Partial information may include aggregated load and topology information at each subnet, and local information, e.g., the total number of wavelengths used at wavelength converters.

Providing state information is a basic functionality of network management. Traditional network management systems intend to obtain as complete state information as possible [3]. But future IP-WDM networks may have hundreds of links with each link supporting hundreds of wavelengths [4]. This would result in a huge amount of state information for networks without wavelength converters. For instance, let H be the number of links within each subnet, F be the number of wavelengths supported per link at each subnet, and L is the number of subnets, the total amount of information about wavelength usage is in the order of FHL. When $F=200$, $H=250$ and $L=10$, the number of states is half a million. Storing, and updating even a fraction of such a large number of states may result in an undesirably large amount of management traffic. Therefore, it may be prohibitive to manage a large network using complete state information.

Using partial management information is also a requirement of multi-vender services. A light-path may trespass multiple administrative domains (sub-networks) run by different service providers. A service provider may prefer to exchange only minimal information to other managed domains rather than share complete state information of its own.

In fact, it has been the experience today in the Internet that network managers of different administrative domains are extremely reluctant and rarely share detailed network state information of their subnets with others. Therefore, inter-domain subnets are like network clouds to a service provider (see Figure 1 for illustration) [5]. Light-path assessment may have to use partial information on network clouds since complete management information is infeasible to obtain across domain boundaries.

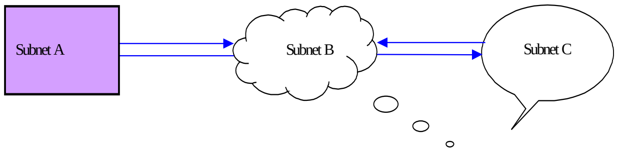

Figure 1. Network Clouds

Intuitively, management information should be considered as scalable if it could be applied to large optical networks with a small loss in performance and without a significant increase in management complexity. As mentioned above, the abundant resources in future optical networks, i.e., the total number of wavelength channels available within the network, makes it prohibitive to manage the network using complete information about wavelength usage. Therefore, complete information should be considered as nonscalable. It may be inevitable to manage the network using partial management information that is scalable. One fundamental issue is what performance can possibly be achieved given the partial information. More specifically, the related issues are: (1) what is the best performance of light-path assessments with partial information? (2) what is the trade-off between the performance and the scalability of network-management information?

Little has been studied on the trade-off between performance and the amount of management information ([6] shows a good example). Intuitively, the performance should be defined in a way to evaluate the sufficiency/insufficiency of the partial information. When complete information is available, the performance would be the best. When the network management information is reduced, the performance would degrade accordingly.

We pose the problem of light-path assessment as a decision problem, and define the performance as the probability of an erroneous assessment. An error occurs when an assessment decision differs from the ground truth (in terms of availability of wavelengths on a given path). The value of the error probability measures a deviation from the optimal performance (with zero error) when the complete information is available, and thus quantifies the sufficiency/insufficiency of the partial management information. We define the scalability of management information using the growth rate of the management information that needs to be maintained with respect to the amount of resources within the network network. For a network having L subnets with each subnet having H links, and with each link supporting F wavelength channels, the total amount of resources is FHL. If the growth rate of the partial information is less than the linear growth rate of the resources, the partial information is considered as scalable.

We investigate a simple network (bus) topology in this work to study the performance of one type of partial management information. The network bears the same spirit as that in Figure 1, i.e., wavelength converters are only located at the boundaries of, but not within, each subnet. The partial information we consider includes (a) aggregated information on network load (and topology) within subnets, and (b) local state information at wavelength converters. The aggregated information serves as model parameters of wavelength usage, and the local information corresponds to random states or observations obtained locally at domain boundaries. For a bus topology with F available wavelengths at each subnet and L subnets, the total amount of the partial information is $O(\log F)$. This is much less than complete state information about wavelength usage, which is the same as the total amount of resources availabe in the network. For this type of partial information to be scalable, it is expected that a small probability of error can be obtained for most of the network-load conditions when it is used for light-path assessment.

To investigate the performance-scalability trade-off, we consider the Bayes decision rule, which results in the Bayes probability of error, which is the best performance achievable given the partial information. We show that the Bayes error is bounded by min $\{P_b, 1-P_b\}$, where P_b is the blocking probability of a light-path. This links our performance measure with a metric commonly-used for WDM networks [7] [8] [9]. The (Bayes) probability of error can then be investigated using different models for blocking probability. We first adopt an independent model corresponding to intra-domain calls. We then extend the independent model to a dependent model to include inter-domain calls. We show that the upper bound is tight when F is large for the independent model. Furthermore, one important characteristic of the performance-scalability trade-off is a ``threshold effect''. That is, there exists a threshold for the load. When the load is close to the threshold value, the blocking probability makes a sharp transition from 0 to 1 for F to be sufficiently large. The probability of error decreases, accordingly, exponentially to zero for most of the load conditions. This suggests that a small loss in performance may be traded off with a large saving in network management information.

The paper is organized as follows. Section II summarizes the prior work. Section III provides problem formulation. Section IV presents Bayes decision theory, and an upper bound of the best performance (the Bayes error) that can be achieved given the partial information. Sections V and VI investigate the best performance using independent model and the dependent model respectively. Section VII concludes the paper.

II. Related Work

Various schemes have been proposed for managing IP-WDM networks based on different amount of management information. One extreme case is to use complete state information on establishing connections [10]. This approach, as discussed earlier, is not feasible for dynamically setting up connections or flows for large networks. Another extreme case is to manage sub-networks as separate entities [11]. But the corresponding performance (i.e., the correctness of an assessment) can be poor due to lack of information. An intermediate approach is proposed to use partial information-exchange among network clouds (subnets) [12]. The idea of using partial information is also investigated in other related applications such as network survivability [13] [14], path protection [15], and path assessment [16]. These works have a different focus, which is mostly on developing approaches to manage networks using partial information. They motivate this work to investigate the achievable performance given scalable partial management information.

Another way to obtain information from network clouds is through probing MPLS paths [17]. Probing methods, however, are intrusive, and thus may be difficult to use for inter-domains due to security reasons.

Wavelength converters (optical or electronic) have been considered as network elements in designing WDM architecture for improving network utilization [18]. Sparsely-allocated wavelength converters, when only reside on the boundaries of inter-connected rings, have been found to be almost always sufficient to achieve a desired utilization gain [19]. The use of wavelength converters has also been conjectured to result in simplified network management systems [18], due to their ability to reduce the state information. This motivates us to consider a natural network architecture where wavelength converters are located at the boundaries of subnets (administrative domains).

Prior investigations in other related areas are beneficial to this research. In particular, inaccurate or aggregated [6] information has been investigated in the context of QoS routing for IP network. Commonly used aggregated information is topology aggregation [20] that can be regarded as a summarized characterization of a subnet. Local information is considered in [21] for QoS routing in IP networks. The main focus of aforementioned work is on managing existing (IP) rather than IP-WDM networks, and the performance-scalability trade-off has not been investigated widely.

III. Problem Formulation

A. Network Architecture

We consider assessing wavelength availability on one light-path for an end-end call from *S* to *D* as shown in Figure 2. Wavelength converters are located at the boundaries of a one-dimensional subnet and there are *L* subnets on a given path. Each subnet has *H* hops and *F* usable wavelengths. Such network architecture, although simple, bears the spirit of that in Figure 1. For simplicity, we assume the subnets are identical, although our approach and analysis apply to sub-networks with different topology (*H*), resource (*F*) as well as the load conditions.

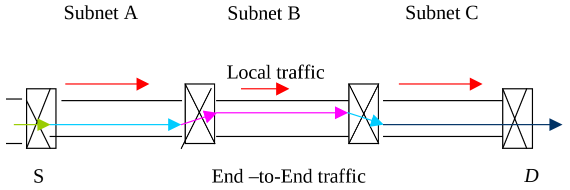

Figure 2: Network Architecture

B. Partial Management Information

The partial information we consider consists of aggregated and local information. The aggregated information characterizes the average behavior of each network cloud so that detailed network states within the subnet are not needed. To be more specific, the aggregated information on network cloud *i* (subnet *i*) is: $A_i = (F_i, H_i, \rho_i, \pi_i)$, where F_i and H_i represent network resource (the number of wavelengths) and topology. ρ_i is the probability a wavelength is used on a link, which is the load information aggregated over all detailed states about wavelength usage within a subnet. π_i can be other parameters related to wavelength usage. For simplicity of analysis, we assume that all *L* subnets have the same aggregated information. Then *A* becomes $A = (F, H, L, \rho, \pi)$.

In practice, the aggregated information may be estimated through measurements, which may deviate from true information, and thus introduce additional information loss. For simplicity, we regard aggregated parameters to be accurate. These parameters may also change with time but at a much larger time scale than the connection dynamics and could thus be regarded as nearly static. The local information corresponds to the number of wavelengths used at the first hop of each subnet, which is readily available at the wavelength converters. To be more specific, the local information corresponds to observations (states) at the wavelength converters is given as $X=(N_1, N_2,\ldots, N_L)$, where N_i is the number of wavelengths used at the first hop of the *i*th subnet. Such local information is changing with setup and teardown of connections, and thus can be considered as random variables.

Such local information is informative due to the wavelength continuity constraint within a subnet. For instance, if nearly all wavelengths are used at the first hop of a subnet, we can infer that the load is high and there may not be any wavelength available within the subnet to support an additional end-end call. Likewise, the aggregated information is informative since it characterizes the average load in a subnet. But the aggregated and local information is still incomplete in determining network states, resulting in possibly erroneous wavelength assessments.

C. Decision Problem and Performance

We pose the light-path assessment problem as a decision problem. We define ω as a decision variable. $\omega = 1$ if there is one end-to-end wavelength continuous path across subnets for the connection request; and $\omega = 0$ otherwise. The problem of light-path assessment is to decide on ω given the partial information. Then the performance of light-path assessment can be defined as the probability of erroneous decisions.

Definition 1: The probability of error P_e is defined as the probability that the assessment decision is different from the ground truth (in terms of availability of wavelengths on a given path).

Let D be the decision region on the management information X for $\omega = 1$; $\overline{D}$ is the decision region for $\omega = 0$. That is, if the observation X falls in D ($\overline{D}$), the decision should be $\omega = 1\,(\omega = 0)$. We then have the probability of error

$$P_e = P(X \in D,\ \omega = 0) + P(X \in \overline{D},\ \omega = 1) \qquad (1)$$

P_e characterizes the average performance given the partial information. The validity of such a performance measure can be understood through Figure 3. When the complete information is available, no error is made in assessing wavelength availability, and the performance is the best (i.e., zero error). When no information is available, decisions can only be made based on random guessing, and the performance is the worst (i.e., 50% error). The value of P_e thus measures a deviation from the optimal performance (zero error) when the complete information is available, and thus quantifies the sufficiency/insufficiency of management information available.

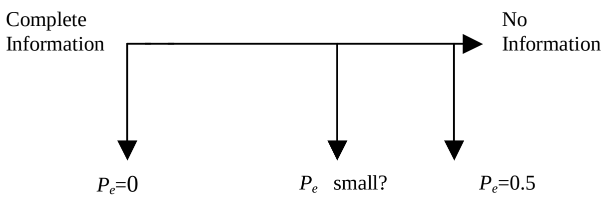

Figure 3: Performance vs. amount of management information

A question is whether it is possible to use scalable management information at the cost of a small number of incorrect decisions.

D. Scalability of Information and Performance

To quantify the scalability of partial information, we define it as follows.

Definition 2: Let R be the total amount of resources within the network, i.e., the total number of wavelengths channels supported by the network; Let Q_P be the amount of partial information about wavelength usage that needs to be maintained. If $Q_P = o(R)$, then the partial information is deemed scalable, and is considered as nonscalable otherwise.

Consider the network shown in Figure 2. The amount of wavelength channels supported by each link is F. The amount of resources within each subnet is FH, therefore,

$$R=FHL. \qquad (2)$$

Generally $L << F$, $L << H$. Clearly, complete information is non-scalable according to definition 2.

For the partial management information considered here,

$$Q_P = L\log(F) + Q_A, \qquad (3)$$

where Q_A is amount of the aggregated information A. $\log(F)$ is the amount of information needed to characterize local states at one subnet. Q_A is generally small, i.e. $Q_A = O(L)$, and changes slowly with time. Then the amount of partial information is in the order of *log F*, which is much less that of the complete management information especially when the number of wavelengths is large. Therefore, this type of partial information is scalable.

IV. Optimal performance using Bayes rule

A. Bayes Error

With partial management information, assessment schemes based on Bayes decision rule [22] achieve the best performance achievable. Given a set of observations $X=(N_1,N_2,...,N_L)$, the Bayes rule is to decide $\omega =1$ if $P(\omega =1 | X=x) > P(\omega =0 | X=x)$, and $\omega =0$ otherwise, where $P(\omega =1 | X=x)$ and $P(\omega =0 | X=x)$ is *a posteriori* probability given observation *x*.

$P(\omega =0 | X=x) = P(\omega =1 | X=x)$ corresponds to the decision boundary, which divides the space (*X*) into two regions, *D* for deciding $\omega =1$ and $\overline{D}$ for deciding $\omega =0$. The Bayes error is the average probability of error, where

$$P_e = P(X \in D, \omega =0) + P(X \in \overline{D}, \omega =1) \quad (4)$$

B. Centralized Light-path Assessment

Such a Bayes rule essentially corresponds to an optimal centralized assessment scheme. Imagine a fictitious central manager, collecting partial information from all subnets. The aggregated information could be polled from each subnet by the central manager at a relatively larger time-scale than the flow dynamics. The local observations *X* could be collected by the central manager at a small time scale. The central manager would then perform the Bayes rule to assess wavelength availability.

This centralized scheme is only conceptual, and is used in this work for performance-scalability analysis rather than a practical solution. Centralized assessment may not be feasible for large optical networks because each subnet may belong to different administrative entities. Thus a distributed light-path assessment scheme may be a necessity. However, distributed assessment schemes result in further information loss due to the decentralization. Therefore there is a need to understand the best performance achievable using the partial information. Such best performance would then serve as a basis for evaluating the performance of sub-optimal yet practical schemes.

C. Bayes Error and Blocking Probability

Although the Bayes error characterizes the optimal performance, it is difficult to evaluate because the decision regions and the corresponding probabilities are hard to obtain. Therefore, we derive an upper bound for the Bayes error. Our intent is to relate such a bound with a commonly used network measure such as blocking probability. Such a relation may provide intuition on how error decisions are related to the load (ρ) and resource (*F*) of subnets.

For clarity, we define the blocking probability as given in [7].

Definition 3: The blocking probability P_b is defined as the probability that there does not exist a wavelength continuous path to support an end-to-end connection.

A relation between the Bayes error P_e and the blocking probability P_b can be derived.

Theorem 1: $0 \le P_e \le \min\{P_b, (1-P_b)\}$

The proof of the theorem can be found in [23].

Intuitively, the upper bound $\min\{P_b,(1-P_b)\}$ can be understood as follows. Consider the following decision rule: If the blocking probability of the network is $P_b > ½$, one can reject all connection requests. Otherwise if $P_b < ½$, one can simply accept all requests. This decision rule will have $P_e = \min\{P_b,(1-P_b)\}$. Since Bayes rule uses local observation *X* as the additional information for light-path assessment in an optimal fashion, a better performance should be achieved. That is, the Bayes error should be bounded by $\min\{P_b,(1-P_b)\}$. Furthermore, with Bayes rule, new connection requests are not always being accepted or always being blocked before the aggregated load information in each subnet is updated. The upper bound shows that the probability of error is small if the blocking probability is close to 1 or 0.

This theorem suggests a feasible way to estimate the Bayes error, which is through the blocking probability. In addition, the bound is obtained independent of a specific model of the blocking probability. The analysis can be conducted using different models. We begin evaluating the error bound using independent model.

V. Probability of Error under Independent Model

A. Independent Model

We adopt a simple model in [7], which assumes independent wavelength usage within a subnet and among wavelengths. Then the corresponding aggregated information is $A = (\rho, F, H, L)$, where ρ is the probability that wavelength is used on one link. The local observation is $X=(N_1, N_2, \ldots, N_L)$ as defined in Section III. Due to the independent assumption, the Ni's are independent random variables.

The probability that there is one end-to-end wavelength continuous path is

$$P_{ai} = (1-(1-(1-\rho)^H)^F)^L \qquad (5)$$

where the subindex *ai* means acceptation based on independent model. Therefore, the blocking probability for an end-to-end call is,

$$P_{bi} = 1-(1-(1-(1-\rho)^H)^F)^L \qquad (6)$$

With the independent model, the *a posteriori* probability is

$$f(X) = P(\omega = 1 | X) = \prod_{k=1}^{L} (1-(1-(1-\rho)^{H-1})^{(F-N_i)}) \qquad (7)$$

where $i = 1,2,\ldots,L$. This expression means that: if N_i wavelengths are used at the first hop in subnet *i*, one only needs to decide whether there is one wavelength continuity path at these *H-1* hops from F-N_i candidate wavelengths. Then $1-(1-(1-\rho)^{H-1})^{(F-N_i)}$ is the probability that there is a continuous wavelength at the *i*th subnet given N_i, and the product is the probability that the connection request for an end-to-end call can be supported.

B. Numerical Analysis

The Bayes error is:

$$P_e = P(f(X) \geq 1/2, \omega = 0) + P(f(X) < 1/2, \omega = 1) \qquad (8)$$

Thus P_e has no close form. Figure 4 plots the blocking probability P_{bi}, (given in Equation (6)) vs. the load (ρ) for F= 10, 40, 120, H=5, L=3. An interesting result is that there is a threshold phenomenon on P_{bi}. When ρ is below the threshold value (e.g. about at ρ=0.5 for F=120), P_{bi} remains close to 0. When ρ is above the threshold value, P_{bi} increases to 1 rapidly. With a larger value of F, the value of the threshold will increase and the transition of P_{bi} from 0 to 1 will be sharper.

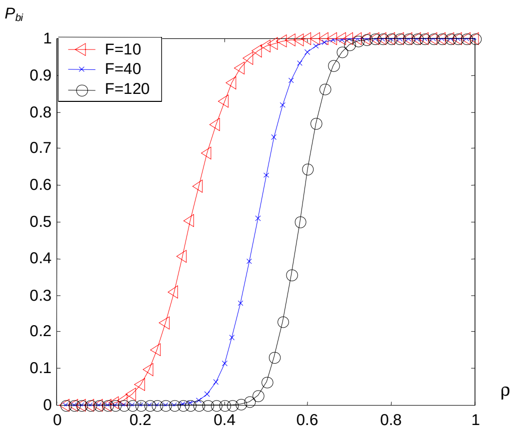

Figure 4. Load (ρ) vs. blocking probability (P_{bi}) for F=10, 40, 120, H=5, L=3.

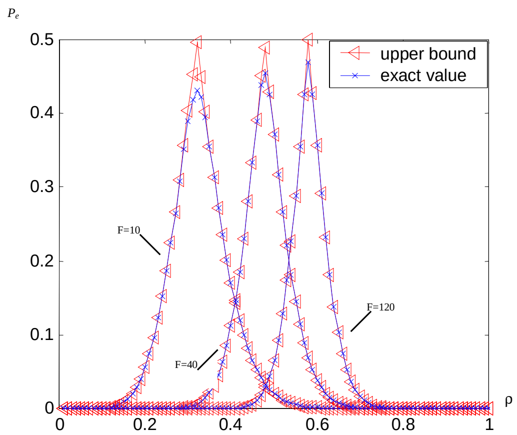

Figure 5 load (ρ) vs. the exact value and the upper bound of P_e for F=10, 40, 120, H=5, L=3.

This shows that under most load conditions, we either have a very small or a very large blocking probability, both of which result in a small probability of error. Therefore, based on Theorem 1, we can conclude that under most load conditions the probability of error for light-path assessment using partial information is small under independent model. Figure 5 confirms this by comparing the upper bound and the exact value of P_e for F= 10, 40, 120, H=5, L=3.We can see that when the load is close to the threshold, the value of P_e increases to the maximum value exponentially. Otherwise, it is small. Furthermore, with a larger value of F ($F \geq 40$), the upper bound is becoming very tight.

C. Special cases

To quantify the decay rate of the upper bound for large F, we consider special cases of low and high load, which correspond to two parts of P_b below and above the threshold. Through algebraic manipulations, we can find that:

when the load is light, i.e., $F >> \frac{1}{(1-\rho)^H}$, $0 \le P_e \le 2L[1-(1-\rho)^H]^F$. (9)

when the load is heavy, i.e., $F \le \frac{1}{(1-\rho)^H}$, $0 \le P_e \le 2L(1-\rho)^{FH}$. (10)

These results suggest that the performance trade-off is a small probability of error that decreases exponentially with respect to the number of wavelengths F under at least moderate and high network load.

VI. Probability of error Under Dependent Model

The above independent model fails to capture the inter-domain calls, which extend beyond one subnet. In future optical networks, a significant percentage of the traffic may be transient flows passing through subnets. Therefore, it is important to take the load correlation among subnets into consideration while estimating the probability of error. Dependent models for blocking probability in a bus have been investigated in [7] [8] [9]. However, in [7], the study is restricted to having wavelength converters installed at each node, while the network architecture as shown in Figure 2 is essentially a bus-network with sparsely-allocated wavelength converters. More accurate dependent models for the blocking probability on such a topology can be found in [8] [9]. However, both models are very complex. Here we extend the dependent model in [7] to obtain a relatively accurate and tractable dependent model to analyze the probability of error.

A. Dependent Model

To capture the dependence on traffic flows among subnets, we assume that there are two types of calls supported by the network. One corresponds to local calls with hop-length equal to 1 as assumed in the independent model. The other type of call corresponds to inter-domain calls (Figure 6). Generally, inter-domain calls can originate and/or terminate anywhere at a network. But for simplicity of analysis, we impose the following assumptions.

(1) The inter-domain calls originate and exit only at edge wavelength converters.
(2) If a wavelength is not used for an inter-domain calls in one subnet, it is used for inter-domain call in the next subnet with probability P_n.
(3) If a wavelength is used for one inter-domain call in one subnet, this inter-domain call will exit the current subnet with probability P_l, and will continue to the next subnet with probability $1-P_l$.
(4) If a wavelength is used for an inter-domain call in one subnet, and it is released at the edge OXC of this subnet. It is used for inter-domain calls with probability P_n in the next subnet.
(5) If an inter-domain call continues to the next subnet, it will use the same wavelength.
(6) In each subnet, a wavelength is used for a local call in a link with probability ρ_1, and for inter-domain call with probability ρ_2. The probability that a wavelength is used for either a local or an inter-domain call is $\rho = \rho_1 + \rho_2$.

The dependent model captures the link correlation across subnets due to inter-domain calls, and is thus more accurate than the independent model. We are aware that it is limited to assume that the inter-domain calls can only enter or exit at the boundaries between subnets. However, such a tractable model provides insights on how inter-domain calls contribute to performance and complexity trade-off. A more realistic model is to be investigated in subsequent work.

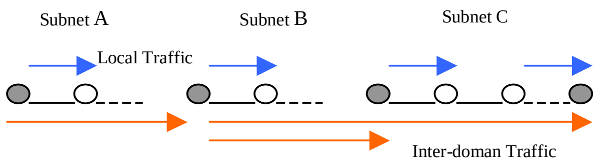

Figure 6: Inter-domain and Local calls

B. Blocking Probability

To derive the blocking probability, we define $\alpha = \rho_2 / \rho$, which characterizes the percentage of wavelengths used for inter-domain calls. Then the independent model is just one special case of the dependent with $\alpha = 0$ ($\rho_2 = 0$). From assumptions in section VI. A., we have,

(1) P(wavelength w_j is used for inter-domain call in subnet i| w_j is not used for inter-domain call in subnet i-1)=P_n
(2) P(wavelength w_j is used for inter-domain call in subnet i| w_j is used for inter-domain in subnet i-1)=$P_nP_l+(1-P_l)$.

Therefore, $$\rho_2 = (1-\rho_2)P_n + \rho_2[P_n P_l + (1-P_l)] \quad (11)$$

It follows that $$P_n = \frac{\rho_2 P_l}{1-\rho_2(1-P_l)}. \quad (12)$$

Define $I_i = 1$ if there is one wavelength continuous path within subnet i; and $I_i = 0$, otherwise. Then a decision that there are wavelengths available for an end-end call ($\omega = 1$) is equivalent to I_i=1 for all i. Let M_i be the number of inter-domain connections in subnet i. Then the blocking probability can be expressed as:

$$\begin{aligned} P_b &= 1 - \sum_{M_1, M_2, \ldots M_L} \{ P(I_1 = 1, I_2 = 1, \ldots, I_L = 1 | M_1, M_2, \ldots, M_L) P(M_1, M_2, \ldots, M_L) \} \\ &= 1 - \sum_{M_1, M_2, \ldots M_L} \{ P(I_1 = 1 | M_1) P(M_1) P(I_2 = 1 | M_2) P(M_2 | M_1) \ldots P(I_L = 1 | M_L) P(M_L | M_{L-1}) \}, \end{aligned} \quad (13)$$

where

$$P(I_i = 1 | M_i) = 1 - [1 - (1-\rho')^H]^{(F-M_i)}, \quad (14)$$

with $\rho' = \rho_1 / (1-\rho_2)$. ρ' is the probability that a wavelength is used for local calls conditioned on that it is not used for inter-domain calls.

Let M_{1i} be the number of inter-domain calls in the i-th subnet that continue to the next subnet. Since only the M_{1i} cause the load dependence between the two subnets, we have

$$P(M_i | M_{i-1}) = \sum_{M_{1i-1}=0}^{M_{i-1}} P(M_i | M_{1i-1}) P(M_{1i-1} | M_{i-1})$$

where

$$P(M_{1i-1} = m | M_{i-1} = k) = \binom{k}{m} P_l^{\,k} (1-P_l)^{k-m} \quad \text{for } 0 \le m \le k \le F, \quad (15)$$

$$P(M_i = h | M_{1i-1} = m) = \binom{F-m}{h-m} P_n^{\,(h-m)} (1-P_n)^{F-h} \quad \text{for } 0 \le m \le h \le F \quad (16)$$

Plugging Equations (14), (15) and (16) into Equation (13), P_b can be computed.

C. Probability of Error

We begin evaluating the performance by considering the probability of error. Again, we assume that all subnets have identical aggregated information. Under the dependent model, the aggregated information A is A=(ρ_1, ρ_2, P_l, F, H, L). Local information is the same as that used for independent model, which is =($N_1, N_2, \ldots, N_L$).
Then *a posteriori* probability used in Bayes rule for X=x is:

$$f(X) = P(\omega = 1 | X) = \prod_{k=1}^{L} (1 - (1 - (1-\rho')^{H-1})^{(F-N_i)}) \quad (17)$$

Such a posterior probability has a similar form to that of the independent case in Equation (7).

The probability of error thus has the same form as in Equation (8). But due to inter-domain calls, the local observations (Ni's) at wavelength converters are now dependent random variables. Therefore, the Bayes error is difficult to derive, we turn to study the upper bound based on the blocking probability P_b.

D. Numerical Analysis

The blocking probability does not have a close-form expression either but can be evaluated numerically. Figure 7 plots P_b vs. ρ for F=120, H=5, L=3, α =0, 0.6, 0.9, P_l=0.2. The figure shows that ρ has a similar "threshold effect" on the value of P_b to that in the independent model. In additional, the threshold is increasing with α , which is defined as the percentage of usage of wavelengths for inter-domain calls. When $\alpha = 0$, the dependent model is reduced to the independent model, and the threshold has the lowest value. This, intuitively, is due to the fact that dependence of inter-domain calls reduces the blocking probability for a given load ρ . Figure 8 plots P_b vs. ρ for F=20, 40, 120, H=5, L=3, α =0.6, P_l=0.2. The figure shows that the threshold is increasing with the number of wavelengths F. This is due to the fact that the more wavelengths, the smaller the blocking probability for a given load. Meanwhile, P_b seems to experience such a transition for all F consistently. The sharpness of the transition also seems to increase with respect to F, suggesting an asymptotic behavior of the blocking probability for a large F. Figure 9 plots the upper bound for the probability of error from Figure 6 using Theorem 1. The figure shows that the P_e is small under most load conditions.

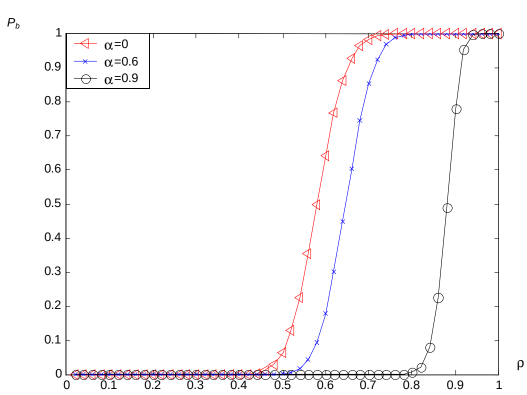

Figure 7. The load (ρ) vs. blocking probability (P_b) for F=120, H=5, L=3, α =0, 0.6, 0.9, P_l=0.2

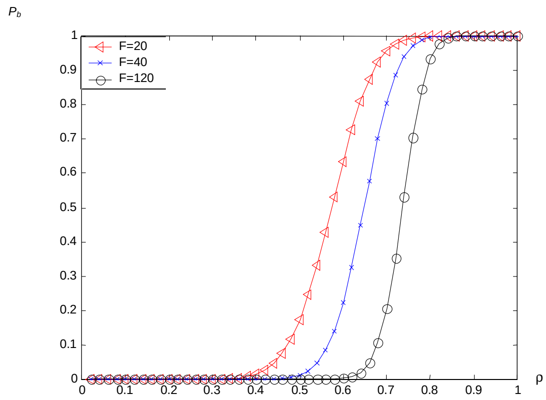

Figure 8. The load (ρ) vs. blocking probability (P_b) for F=20, 40, 120, H=5, L=3, α =0.6, P_l=0.2

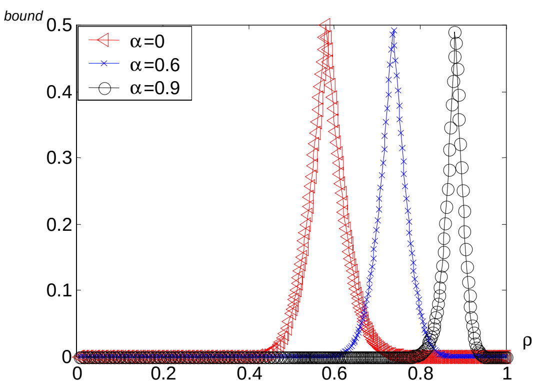

Figure 9. The load (ρ) vs. the uppder bound of P_e for F=120, H=5, L=3, α =0, 0.6, 0.9, P_l=0.2

E. Special Cases

A question rises why the threshold effect persists for both independent and dependent models. We investigate this question by considering special cases when the number of wavelengths available is large using Gaussian approximation. We derive analytical form of the blocking probability for this special case.

E.1 *Gaussian* Approximation

An important step is to approximate the joint probability of the local states (N_i's) at wavelength converters. When the number of wavelengths F is large (and L is small), local states at wavelength converters, $X = (N_1, N_2, \ldots, N_L)$ are jointly Gaussian random variables. (with a probability 1-$O(L/\sqrt{F}$) [24]). Such a Guassian distribution can be completely characterized by the means, variances, and covariances of Ni's. Specifically, all N_i's are random variables with the same mean μ and the same variance σ^2, where

$$\mu = F\rho \qquad (18)$$

and

$$\sigma^2 = F\rho(1-\rho). \qquad (19)$$

The covariance C_{ij} between N_i and N_j for $i \neq j$ characterizes the dependence between two subnets, where

$$C_{ij} = E[N_i N_j] - \mu^2.$$

Such dependence can be further characterized through partitioning N_i and N_j into different components,

$$N_i = N_{ii} + M_{it} + M_{ij} \qquad (20)$$

$$N_j = N_{jj} + M_{jo} + M_{ij}, \qquad (21)$$

where,

N_{ii} is the number of wavelengths occupied by local calls at the first hop of the *i-th* subnet.

N_{jj} is the number of wavelengths occupied by local calls at the first hop of the *j-th* subnet.

M_{it} is the number of wavelengths in the *i-th* subnet occupied by inter-domain calls that terminate before entering the *j-th* subnet.

M_{jo} is the number of wavelengths in the *j-th* subnet occupied by inter-domain calls that originate after the *i-th* subnet.

M_{ij} is the number of wavelengths used in the *i-th* subnet occupied by inter-domain calls that extend from the i-th subnet to the *j-th* subnet.

It is the common factor M_{jj} in N_i and N_j that causes the dependence between two subnets. Therefore, it is easy to derive that,

$$C_{ij} = \mathrm{var}[M_{ij}] \qquad (22)$$

Furthermore, the average number of inter-domain calls is $F\rho_2$. The probability for an inter-domain call in subnet *i* to extend to subnet *j* is

$$P_{ij} = 1 - P_l \sum_{k=0}^{j-i+1} (1-P_l)^k = (1-P_l)^{j-i} \qquad (23)$$

Then due to the binomial nature of M_{ij},

$$C_{ij} = F\rho_2 P_{ij}(1-P_{ij}) \qquad (24)$$

Then the correlation coefficient is $C_{ij} \big/ \sigma_{N_i}^2$, which is

$$\rho_{ij} = \frac{\rho_2 P_{ij}(1-P_{ij})}{\rho(1-\rho)} \qquad (25)$$

E.2 *Relation* to Independent Case

Through Taylor Expansion, we have the following Theorem.

Theorem 2: The non-blocking probability of the dependent model can be expressed as

$$P_a = P_{ai} + \eta + o(\max\{\rho_{ij}\}), \qquad (26)$$

where $P_{ai} = (1-(1-(1-\rho)^H)^F)^L$ is the non-blocking probability of the independent case given in Equation (5), and $\eta = [1-(1-(1-\rho)^H)^F]^{L-2} q_{ij} \sum_{1\le i\le j\le L} \rho_{ij}$ is the first order term in the Taylor Expansion. Detailed derivations can be found in [23].

It is found that:

(1) When $P_l = 1$ or 0, $\rho_{ij} = 0$, which corresponds to the case when all inter-domain calls lasts for one subnet or all inter-domain calls are end-to-end. Therefore, $P_a = P_{ai}$ and the dependent model reduce to the independent model.

(2) When P_l is large (e.g. $P_l \ge 0.9$), which corresponds to the case that most of the inter-domain calls leave that the current subnet, we have $\sum_{1\le i\le j\le L} \rho_{ij} = O(1-P_l)$ and $\eta = O(1-P_l)$. Hence $P_a = P_{ai} + O(1-P_l) + o(\max\{\rho_{ij}\})$. The non-blocking probability in the dependent model is just that of the independent model plus a small perturbation.

(3) When P_l is small (e.g., $P_l \le 0.1$), which corresponds to the case that most of the inter-domain calls will continue to the next subnet, the value of $\sum_{1\le i\le j\le L} \rho_{ij}$ could be very large. Therefore, η becomes the dominant term in equation (26). Since $\eta = [1-(1-(1-\rho)^H)^F]^{L-2} q_{ij} \sum_{1\le i\le j\le L} \rho_{ij}$, $[1-(1-(1-\rho)^H)^F]^{L-2}$ has a similar form to P_{ai}, we can expect that there exists a similar “threshold effect” for the dependent model as for the independent model.

The analysis here shows why the threshold effect persists for both independent and dependent model when F is large for some typical cases.

VII. Conclusion

In this paper, we investigate network management information for light-path assessment across administrative domains (subnets). Our focus has been on studying the best performance achievable in assessing wavelength availability using scalable partial management information. The partial management information includes aggregated information of subnets, and local information from wavelength converters on the network-domain boundaries. The amount of partial information is thus much reduced compared with the detailed and complete state information. We have shown that the performance and scalability trade-off can be studied in the context of Bayes decision theory, where the optimal performance is the Bayes probability of error. Such an optimal decision essentially characterizes what can possibly be achieved using the partial information in an average sense.

We have shown that such an optimal performance is upper-bounded in terms of the blocking probability of a light-path. We have evaluated the blocking probability and the upper bound using both independent and dependent models of wavelength usage. Our study reveals an interesting phenomenon that, when the number of wavelengths is large, the blocking probability transits from 0 to 1 rapidly. This results in a Bayes error negligibly small for most of the network load conditions. Such results suggest that a small loss in performance (the Bayes error) may result in a large saving in network management information. That is, the abundant network resource, which is the large number of wavelengths of future WDM networks may make it possible to reduce the amount of network management information while achieving a good performance. Our current efforts are in extending this work to a more general network topology, and to develop practical distributed assessment schemes that can hopefully achieve good optimal performance.

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