# On the Scalability of Network Management Information for Inter-Domain Light-Path Assessment

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***Abstract***— We investigate the necessary amount of network management information for light-path assessment to dynamically set up end-to-end light-paths across administrative domains in optical networks. Our focus is on the scalability of partial management information. We pose light-path assessment as a decision problem, and define the performance as the Bayes probability of an erroneous decision. We then characterize the scalability of management information as its growth rate with respect to the total resources of the network, to achieve a desired performance. Scalability is achieved if the management information needed is only a negligible fraction of the total network resources. Specifically, we consider in this work one type of partial information that grows only logarithmically with the number of wavelengths supported per link. We derive an upper bound for the Bayes error in terms of the blocking probability when a new call is presented at the entrance of the network. We evaluate the upper bound using both independent and dependent models of wavelength usage for intra- and inter-domain calls. Our study shows that there exists a "threshold effect": The Bayes error decreases to zero exponentially with respect to the load when the load is either below or above a threshold value; and is non-negligible when the load is in a small duration around the threshold. This suggests that the partial information considered can indeed provide the desired performance, and a small percentage of erroneous decisions can be traded off to achieve a large saving in the amount of management information.

**Index Terms— Scalability, management information, light-path assessment, decision theory, Bayes rule, blocking probability.**

## I. Introduction

DYNAMICALLY assessing the quality of light-paths is important to many applications in wavelength-routed optical networks such as on-demand light-path provisioning, protection and restoration. As the light-path quality is a complex measure [1], this work considers a simple quality, which is the wavelength availability on a candidate light-path. The assessment then boils down to determine availability of wavelengths for incoming call requests based on given management information.

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Complete or partial network management information can be used to assess the wavelength availability on a light-path. Complete information corresponds to the detailed states of wavelength usage, i.e. "which wavelengths are used at which links of a network", when there are no wavelength converters in the network. Wavelength converters can reduce state information due to their ability to relax the wavelength continuity constraint. However, it is expected that wavelength converters remain expensive and are thus used mostly on the boundaries of sub-networks [2]. Therefore, generally complete state information involves the detailed wavelength occupancy within a subnet. Partial information includes aggregated load and topology information at each subnet, and local states, e.g., the total number of wavelengths used at wavelength converters.

Providing state information is a basic functionality of network management. Traditional network management systems intend to obtain as complete state information as possible [3]. But future IP-WDM networks may have hundreds of links with each link supporting hundreds of wavelengths [4]. This would result in a huge amount of state information for networks without wavelength converters. For instance, let $H$ be the number of links within each subnet, $F$ be the number of wavelengths supported per link at each subnet, and $L$ be the number of subnets. The total amount of information about wavelength usage is in the order of $FHL$. When $F=200$, $H=250$ and $L=10$, the number of states is about half a million. Storing and updating even a fraction of such a large number of states may result in an undesirably large amount of management traffic. Therefore, it would be prohibitive to manage a large network using complete state information.

Using partial management information is also a requirement of multi-vendor services. A light-path may traverse multiple administrative domains (sub-networks) run by different service providers. A service provider may prefer to exchange only minimal information to other network domains rather than share the complete state information of its own. In fact, it has been the experience today in the Internet that network managers of different administrative domains are extremely reluctant to and rarely share detailed network state information of their subnets with others. Therefore, inter-domain subnets are like unknown network clouds to a service provider [5]. Light-path assessment may have to use partial information on network clouds since it is infeasible to obtain complete management information across domain boundaries.

Therefore, a fundamental issue in light-path assessment is what performance can possibly be achieved given the partial information. Specifically, the related questions are: (1) what is the best performance of light-path assessments with the partial information? (2) What is the trade-off between the performance and the amount of management information maintained for light-path assessments?

We formulate the light-path assessment as a decision problem, and define the performance as the probability of an erroneous assessment. An error occurs when an assessment decision differs from the ground truth (in terms of availability of wavelengths on a given path). The value of the error probability measures the deviation from the optimal performance (with zero error) when the complete information is available, and thus quantifies the sufficiency/insufficiency of the partial management information.

With a large amount of management information, a good performance, i.e., a small error probability, could be achieved but at the cost of management complexity such as signaling and memory overhead. With a small amount of management information, the performance may degrade but with a gain of management simplicity. Thus a trade-off can be made between the performance and the network management information.

The amount of management information needed varies with respect to the size, and the resource of the network. The size can be characterized by the number of links in a subnet and the number of subnets. The resource corresponds to the total number of wavelengths, which is related to the number of users (flows) supportable by a network. Future optical networks may have hundreds of links, each of which supports hundreds of wavelengths. Therefore, the growth rate with respect to those parameters is an important measure of the amount of management information used. In particular, a desirable growth rate should be slower than that of the total resource to be managed in a network.

Combining the performance and the growth rate, we define the scalability of network management information for light-path assessment. Assuming that a given performance is satisfied, i.e., a small probability of error can be achieved; we consider the needed management information as scalable if it grows at a slower rate than the total network resource; and as non-scalable, otherwise. Therefore, the scalability requires that the amount of information used is only a negligible fraction of the total wavelength resources within the network. Hence scalability/non-scalability provides a systematic way to investigate the tradeoff between performance and the management information.

In this work, we study one type of "strongly'' scalable management information, which is only logarithmic ($O(\log F)$) in the number of wavelength supported per link in the network. We investigate a simple network of bus topology to study the scalability of the partial management information. Wavelength converters are only located at the boundaries of, but not within, each subnet. The partial information we consider includes (a) aggregated information on network load and topology within subnets, and (b) local state information at wavelength converters. The aggregated information serves as model parameters of wavelength usage, and the local information corresponds to random states or observations obtained locally at domain boundaries. For a bus topology with $F$ available wavelengths at each link and $L$ subnets, the total amount of the partial information is $O(L \log F)$. This is indeed much less than the total amount of resources availabe in the network (*FHL*). Therefore, the partial information will introduce much less management complexity than complete information.

To evaluate the achievable performance using the partial information, we consider the Bayes decision rule. The Bayes rule results in the best performance achievable given the partial information, which is the Bayes probability of error. We show that the Bayes error is bounded by $\min\ \{P_b,\ 1\text{-}P_b\}$, where $P_b$ is the blocking probability of a light-path. This links the Bayes error with $P_b$, a metric commonly used for WDM networks [6] [7] [8]. The (Bayes) probability of error can then be investigated through the blocking probability based on different traffic models. We first adopt an independent model that corresponds to local calls. We then extend the independent model to a dependent model to include inter-domain calls. One important characteristic of the best performance using the partial information is a "threshold effect'', i.e., there exists a threshold for the load. When the load is close to the threshold value, the blocking probability makes a sharp transition from 0 to 1. The corresponding probability of error remains close to zero for most of the load conditions. This suggests that the partial information could provide desirable performance for light-path assessment. Hence the partial information is scalable and a small loss in performance may be traded off with a large saving in network management information.

The paper is organized as follows. Section II summarizes the prior work. Section III provides the problem formulation. Section IV presents Bayes decision theory, and an upper bound of the best performance (the Bayes error) that can be achieved given the partial information. Sections V and VI investigate the best performance using an independent model and a dependent model respectively. Simulation results are presented in section VII. Section VIII concludes the paper.

## II. Related Work

Various schemes have been proposed for managing IP-WDM networks based on different amount of management information. Complete state information has been used to establish connections [9]. This approach, as discussed earlier, may not be feasible for dynamically setting up inter-domain connections for large networks. In contrast to using complete information, another method is to manage sub-networks as separate entities [10]. The corresponding performance (i.e., the correctness of an assessment) can be poor due to lack of information. An intermediate approach is to use partial information-exchange among network domains [11]. The idea of using partial information is also investigated in other related applications such as network survivability [12] [13] [14], and wavelength routing [15]. These works have a different focus, which is mostly on developing approaches to manage networks using partial information. They motivate this work to investigate the scalability of management information.

Probing methods have been proposed to obtain information from network clouds [16]. Probing, however, is intrusive, and may be impractical for inter-domain light-path assessment because of security reasons.

Wavelength converters (optical or electronic) have been considered in designing WDM networks to improve wavelength utilization [17]. Sparsely-allocated wavelength converters are found to be sufficient to achieve a desired utilization gain sometimes [18]. The use of wavelength converters has also been conjectured to result in simplified network management systems due to their ability to reduce the state information [17]. This motivates us to consider a natural network architecture where wavelength converters are located at the boundaries of subnets (administrative domains).

Prior investigations in other related areas are also beneficial to this research. In particular, inaccurate or aggregated information has been investigated in the context of *QoS* routing for IP networks [19]. Commonly used aggregated information is topology aggregation [20] [21] that can be regarded as a summarized characterization of a subnet. Local information is considered in [22] for *QoS* routing in IP networks. However, the main focus of aforementioned work is on managing existing (IP) rather than IP-WDM networks.

Therefore, the tradeoff between performance and the amount of management information has not been investigated quantitatively. In our prior work [23] [24], we formulated the problem of network management information for light-path assessment based on independent and dependent models of wavelength usage. This work extends the prior work to a more comprehensive setting. We formally define the scalability of management information for light-path assessment, and use both analysis and simulations to investigate the scalability of the information.

## III. Problem Formulation

### A. Network Architecture

We consider assessing wavelength availability for an end-to-end call request from source border node $S$ to destination border node $D$ as shown in Fig. 1. Wavelength converters are located at the boundaries of one-dimensional subnets and there are $L$ subnets on a given path. Here for simplicity, we assume that border nodes with wavelength converters connect two adjacent network domains. Each subnet has $H$ hops and each link supports $F$ wavelengths. Such network architecture, although simple, captures the important characteristic of multi-domain network topologies.

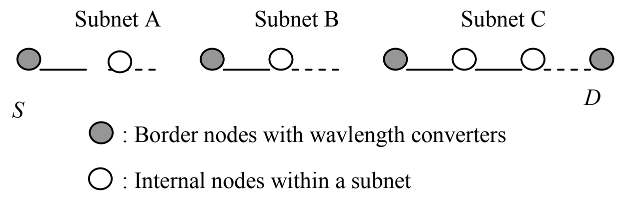


Fig. 1. Network architecture.

### B. Partial Management Information

The partial information we consider consists of aggregated information and local states. The aggregated information characterizes the average behavior of each network domain so that detailed network states within each subnet do not need to be exchanged across domains. The aggregated information is denoted as $A=(A_1, A_2,..., A_L)$, where $A_i$ is the aggregated information on subnet $i$ and $A_i=(F_i, H_i, \rho_i, \pi_i)$. $F_i$ and $H_i$ are the number of wavelengths per link and the number of links at subnet $i$ respectively. $\rho_i$ is the probability that a wavelength is used on a link in domain $i$, which is the load information aggregated over all detailed states about wavelength usage within the subnet. $\pi_i$ are parameters related to wavelength usage. For example, $\pi_i$ could be the parameter that characterizes how wavelengths at each link are used, i.e., the percentage of occupied wavelengths used for inter-domain connections. For simplicity of analysis, we assume that each subnet has the same aggregated information. Then we have $A_i=(F, H, \rho, \pi)$ for all $i$.

In practice, the aggregated information can be estimated through measurements, which may deviate from true parameters, and thus introduce additional information loss. For simplicity, we regard aggregated parameters to be accurate. These parameters may also change with time but at a much larger time scale than the connection dynamics, and could thus be regarded as nearly static.

The local information corresponds to the number of wavelengths used at the first hop of each subnet, which is readily available at the wavelength converters. Specifically, the local information corresponding to observations (states) at the wavelength converters is given as $X=(N_1, N_2,..., N_L)$, where $N_i$ is the number of wavelengths used at the $i$th wavelength converters, i.e., the number of wavelengths used at the first link of domain $i$. Such local information is changing with setup and teardown of connections, and can thus be considered as random variables.

The local information is informative due to the wavelength continuity constraint within a subnet. For instance, if nearly all wavelengths are used at the first hop of a subnet, we can infer that the load is high and there may not be any wavelength available within the subnet to support an additional end-end call. Likewise, the aggregated information is informative since it characterizes the average load in a subnet. But the aggregated and local information is incomplete in determining network states, resulting in possibly erroneous wavelength assessments.

### C. Decision Problem and Performance

We pose the light-path assessment as a decision problem. A decision variable $\omega$ is defined as follows: $\omega = 1$ if there is one end-to-end wavelength continuous path across subnets for the connection request; and $\omega = 0$ otherwise. The problem of light-path assessment is to decide on $\omega$ given the partial information. Then the performance of light-path assessment can be defined as the probability of erroneous decisions.

*Definition 1: The probability of error $P_e$ is defined as the probability that the assessment decision is different from the ground truth (in terms of availability of wavelengths on a given path).*

Let $D$ be the decision region on the management information $X$ for $\omega = 1$; and $\bar{D}$ be the decision region for $\omega = 0$. In other word, if the observation $X$ falls in $D$ or $\bar{D}$, the decision should be $\omega = 1$ or $\omega = 0$ respectively. We then have the probability of error

$$P_e = P(X \in D,\ \omega = 0) + P(X \in \bar{D},\ \omega = 1). \quad (1)$$

$P_e$ characterizes the average performance given the partial information. The validity of such a performance measure can be understood through Fig. 2. When the complete information is available, no error is made in assessing wavelength availability, and the performance is the best (i.e., zero error). When no information is available, decisions can only be made based on random guessing, and the performance is the worst (i.e., 50% error). The value of $P_e$ thus measures the deviation from the optimal performance (zero error) when the complete information is available, and thus quantifies the sufficiency/insufficiency of the management information available. A question is whether it is possible to use partial management information at the cost of a small number of incorrect decisions.

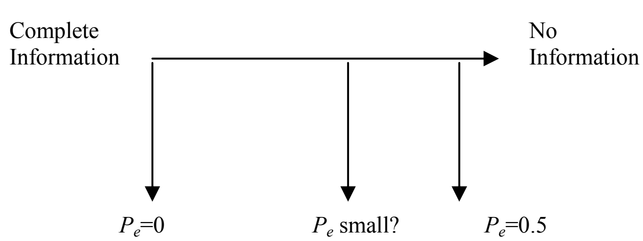


Fig. 2. Performance vs. management information

### D. Scalability of Management Information

We investigate this problem in the context of scalable network management information. Intuitively, there are two important aspects of the scalability: The amount of management information should be sufficiently large to satisfy a given performance, i.e., a small probability of error. Meanwhile, the amount of management information should be small enough to keep network management simple. For instance, it is preferred that the management information needed is just a negligible fraction of the total network resource, e.g. the total number of wavelengths supported in the network. Since the resource varies with respect to the size of a network and the number of wavelengths supported per link, it would be meaningful to characterize the amount of management information required as its growth rate with respect to those quantities. Combining the performance and the growth rate, we formally define the scalability of management information as follows.

*Definition 2: Let $Q_P$ be the amount of management information used for light-path assessment. Let $Q_R$ be the total amount of wavelength resources within the network. If $Q_P$ grows at a slower rate than $Q_R$ with respect to the number of wavelengths per link (F) and the size (HL) of a network, and the corresponding performance of light-path assessment is acceptable under most load conditions, the network management information is scalable; and otherwise, non-scalable.*

This definition essentially means that asymptotically (for a large network with many links and wavelengths each link), the scalable management information is a negligible fraction of the total network resource. That is, $Q_P / Q_R = o(1)$ when $F$ and $HL$ are large.

Consider the network shown in Fig. 1. The number of bits is used to quantify the management information. The detailed states within each subnet are "which wavelengths are used at which link". The total number of possible (binary) states is $2^{FH}$ for each subnet, and $2^{FHL}$ for $L$ subnets. Therefore, complete information satisfies

$$Q_P(complete) = Q_R = FHL. \quad (2)$$

Usually $L << F$ and $L << H$. Clearly, it is non-scalable to use complete management information according to definition 2 even though it will always result in zero probability of error.

The partial management information considered in this work satisfies

$$Q_P(partial) = L\log(F) + Q_A, \quad (3)$$

where $Q_A$ is the number of bits needed to store the aggregated information, which is indexed with $A$. $Q_A$ is generally small, and changes slowly with time. $\log(F)$ is the total number of bits needed to characterize local states at one subnet. Then the amount of partial information is in the order of $\log(F)$, which is much less than that of the complete management information, especially when the number of wavelengths is large. Such partial information can be maintained easily even for a large network.

## IV. Optimal Performance Using Bayes Rule

We now evaluate the best performance of the partial management information to see whether it can provide the desired performance.

### A. Bayes Error

With partial management information, assessment schemes based on Bayes decision rule [25] achieve the best performance. Given a set of local states $X=(N_1,N_2,...,N_L)$, the Bayes rule is to decide

$$\begin{cases}\omega=1, \; if \;\; P(\omega=1 \mid X=x) \geq P(\omega=0 \mid X=x), \\ \omega=0, \; otherwise,\end{cases}$$

where $P(\omega=1\mid X=x)$ and $P(\omega=0\mid X=x)$ is *a posteriori* probability given observation $X=x$. The equality $P(\omega=1\mid X=x)=P(\omega=0\mid X=x)$ corresponds to the decision boundary, which divides the space ($X$) into two regions, $D$ to decide $\omega=1$ and $\bar{D}$ to decide $\omega=0$. The Bayes error is the average probability of error as given in Equation (1).

### B. *Centralized Light-path Assessment*

Such a Bayes rule essentially corresponds to an optimal centralized assessment scheme. Imagine a fictitious central manager, collecting partial information from all subnets. The aggregated information could be polled from each subnet by the central manager at a relatively larger time-scale than the flow dynamics. The local observation $X$ could be collected by the central manager at a smaller time scale. The central manager would then perform the Bayes rule to assess wavelength availability.

This centralized scheme is only conceptual, and used in this work for analysis rather than a practical solution. Centralized assessment may not be feasible for large optical networks because each subnet could belong to different administrative entities. Thus a distributed light-path assessment scheme may be a necessity. However, distributed assessment schemes result in further information loss due to decentralization. Therefore there is a need to understand the best performance achievable using the partial information. Such best performance would then serve as a basis for assessing the performance of sub-optimal yet practical schemes.

### C. *Bayes Error and Blocking Probability*

Although the Bayes error characterizes the optimal performance, it is difficult to evaluate because the decision regions and the corresponding probabilities are hard to obtain. Therefore, we derive an upper bound for the Bayes error. Our goal is to relate such a bound with a commonly used network measure such as blocking probability. Such a relation may provide intuition on how error decisions are related to the load ($\rho$) and wavelength per link ($F$) of each subnet. For clarity, we describe the blocking probability based on [6].

*Definition 3: The blocking probability* $P_b$ *is defined as the probability that there does not exist a wavelength continuous path in each network domain to support an end-to-end inter-domain connection*.

A relation between the Bayes error $P_e$ and the blocking probability $P_b$ can then be derived.

*Theorem 1.* $0 \leq P_e \leq \min\{P_b,(1-P_b)\}$.

The proof of the theorem is given in *Appendix I.* Intuitively, the upper bound $\min\{P_b,(1-P_b)\}$ can be understood as follows. Consider the following decision rule: If the blocking probability of the network is $P_b>1/2$, one can reject all connection requests. Otherwise, if $P_b<1/2$, one can simply accept all requests. This decision rule will have $P_e=\min\{P_b,(1-P_b)\}$. Since Bayes rule uses local observation $X$ as the additional information for light-path assessment in an optimal fashion, a better performance should be achieved. That is, the Bayes error should be bounded by $\min\{P_b,(1-P_b)\}$. The upper bound shows that the probability of error is small if the blocking probability is close to 1 or 0.

This theorem suggests an analytically feasible way to estimate the Bayes error, which is through the blocking probability. In addition, the bound is independent of a specific model of the blocking probability. The analysis can then be conducted using different models.

## V. INDEPENDENT MODEL

### A. *Independent Model*

We first assume independent wavelength usage on different network links and among wavelengths. Such an assumption is equivalent to that all connections within the network are local calls as shown in Fig. 3. Then the corresponding aggregated information is $A=(\rho,F,H,L)$, where $\rho$ is the probability that wavelength is used on one link. The local observation is $X=(N_1,N_2,...,N_L)$ as defined in Section III. Due to the independent assumption, all the $N_i$'s are independent random variables.

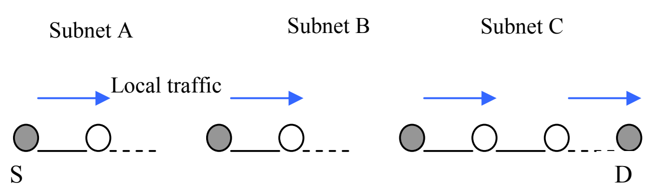


Fig. 3. Local Calls in independent Model

### B. *Bayes Error*

Under the independent model, the *a posteriori* probability is

$$f(X)=P(\omega=1\mid X)$$
$$=\prod_{i=1}^{L}(1-(1-(1-\rho)^{H-1})^{(F-N_i)}), \quad (4)$$

where $i=1,2,...,L$. This expression means that if $N_i$ wavelengths are used at the first hop of subnet *i*, one only needs to decide whether there is a wavelength continuity path at the next *H-1* hops from *F-* $N_i$ candidate wavelengths. Then $1-(1-(1-\rho)^{H-1})^{(F-N_i)}$ is the probability that there is a continuous wavelength at the *i*th subnet given $N_i$, and the product is the probability that the connection request for an end-to-end call can be supported. The Bayes error is:

$$P_e = P(f(X) \geq 0.5, \omega = 0) + P(f(X) < 0.5, \omega = 1). \quad (5)$$

Equation (5) does not have a close form; and we turn to evaluate the upper bound of $P_e$ using the blocking probability of the independent model.

### *C. Numerical Analysis*

Under the independent assumption, the probability that there is one end-to-end wavelength continuous path can be obtained using a model in [6]:

$$P_{ai} = (1-(1-(1-\rho)^H)^F)^L, \quad (6)$$

where the sub-index *ai* means acceptation of a request based on independent model. Therefore, the corresponding blocking probability for an end-to-end call is,

$$P_{bi} = 1-(1-(1-(1-\rho)^H)^F)^L. \quad (7)$$

Fig. 4 plots the blocking probability $P_{bi}$, vs. the load ($\rho$) for *F*= 10, 40, 120, *H*=5, *L*=3. An interesting phenomenon is that there is a threshold on $P_{bi}$. When $\rho$ is below the threshold value (e.g. about at $\rho$=0.5 for *F*=120), $P_{bi}$ remains close to 0. When $\rho$ is above the threshold value, $P_{bi}$ increases to 1 rapidly. With a larger *F*, the value of the threshold increases, and the transition of $P_{bi}$ from 0 to 1 gets sharper.

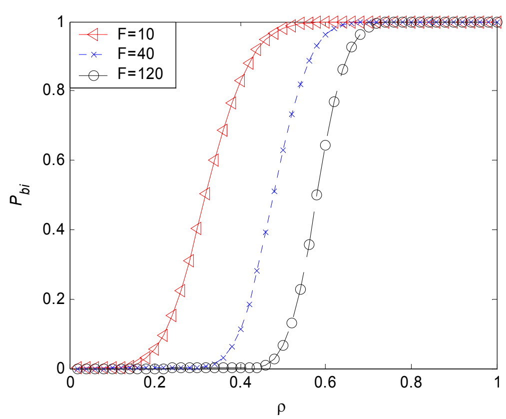


Fig. 4. Load ($\rho$) vs. blocking probability ($P_{bi}$): *F=10, 40, 120, H=5, L=3.*

This shows that under most load conditions, we either have a small or a large blocking probability, both of which result in a small probability of error. Therefore, based on Theorem 1, we can conclude that under most load conditions the probability of error for light-path assessment using partial information is small under independent model. Fig. 5 confirms this by plotting the upper bound of $P_e$ for *F*= 10, 40, 120, *H*=5, *L*=3.We can see that when the load is close to the threshold, the value of $P_e$ increases to the maximum value exponentially; and otherwise, $P_e$ is small.

### *D. Special cases*

To quantify the decay rate of the upper bound for large *F*, we consider special cases of low and high load, which correspond to two parts of $P_b$ below and above the threshold. We can find that:

(*i*) When the load is light, i.e., $F \gg \frac{1}{(1-\rho)^H}$,

$$0 \leq P_e \leq 2L[1-(1-\rho)^H]^F. \quad (8)$$

(*ii*)When the load is heavy, i.e., $F \ll \frac{1}{(1-\rho)^H}$,

$$0 \leq P_e \leq 2L(1-\rho)^{FH}. \quad (9)$$

These results suggest that the performance trade-off is a small probability of error that decreases exponentially with respect to the number of wavelengths per link (*F*) under at least moderate and high network load.

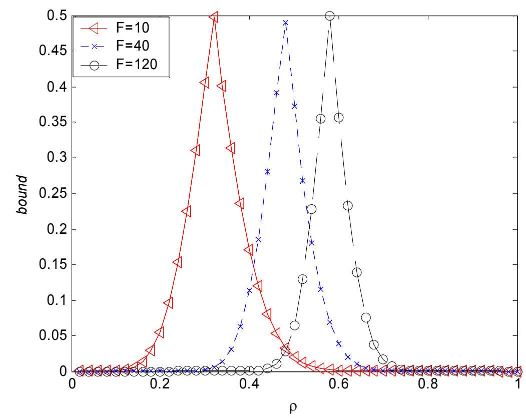


Fig. 5. Load ($\rho$) vs. upper bound of $P_e$: *F=10, 40, 120, H=5, L=3.*

## VI. Dependent Model

The above independent model fails to capture the inter-domain calls, which extend beyond one subnet. In future optical networks, a significant percentage of the traffic may be transient flows passing through subnets. Therefore, it is important to take the load correlation among subnets into consideration when estimating the performance. In this section, we investigate the probability of error by considering both intra- and inter-domain calls.

### *A. Dependent Model*

Dependent models in a bus have been investigated in [6] [7] [8]. However, the study in [6] is restricted to having wavelength converters installed at each node, while the network architecture as shown in Fig. 1 is with sparsely-allocated wavelength converters. More accurate dependent models for the blocking probability on such a topology can be found in [7] [8]. However, both models are complex. Here we extend the dependent model in [6] to obtain a relatively accurate and tractable dependent model for analyzing the probability of error.

To capture the dependence on traffic flows among subnets, we assume that there are two types of calls supported by the network. One corresponds to local calls as assumed in the independent model. The other type of calls corresponds to inter-domain calls (see Fig. 6). Generally, inter-domain calls can originate and/or terminate anywhere at a network. But for simplicity of analysis, we impose the following assumptions:

(*i*) The inter-domain calls originate and exit only at edge wavelength converters.

(*ii*) If a wavelength is not used for an inter-domain call in one subnet, it is used for inter-domain call in the next subnet with probability $P_n$.

(*iii*) If a wavelength is used for one inter-domain call in one subnet, this inter-domain call will exit the current subnet with probability $P_l$, and will continue to the next subnet with probability $1-P_l$.

(*iv*) If a wavelength is used for an inter-domain call in one subnet and is released at the edge OXC of this subnet, it is used for inter-domain calls with probability $P_n$ in the next subnet.

(*v*) If an inter-domain call continues to the next subnet, it will use the same wavelength.

(*vi*) In each subnet, a wavelength is used for a local call in a link with probability $\rho_1$, and for an inter-domain call with probability $\rho_2$. The probability that a wavelength is used for either a local or an inter-domain call is $\rho = \rho_1 + \rho_2$.

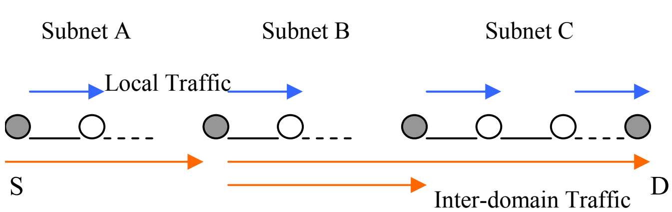


Fig. 6. Inter-Domain calls and local calls.

The dependent model captures the link load correlation across subnets due to inter-domain calls, and is thus more accurate than the independent model. We are aware that it is limited to assume that the inter-domain calls can only enter or exit at the domain boundaries. However, such a model provides understanding of how inter-domain calls contribute to the performance and management information trade-off. A more realistic model is to be investigated in subsequent work.

## B. Bayes Error

We begin evaluating the performance by considering the probability of error. Again, we assume that all subnets have identical aggregated information. Under the dependent model, the aggregated information $A$ is $A = (\rho_1,\ \rho_2,\ P_l,\ F,\ H,\ L)$. Local information is the same as that used for independent model, which is $X = (N_1, N_2, ..., N_L)$. Then the *a posteriori* probability used in Bayes rule is:

$$f(X) = P(\omega = 1 \mid X) = \prod_{k=1}^{L} (1-(1-(1-\rho_c)^{H-1})^{(F-N_i)}), \qquad (10)$$

where $\rho_c = \rho_1/(1-\rho_2)$. $\rho_c$ is defined as the probability that a wavelength is used for local calls given that it is not used for inter-domain calls. Such a posterior probability has a similar form to that of the independent case in Equation (4).

The probability of error thus is the same as in Equation (5). But due to inter-domain calls, the local observations ($N_i$'s) at wavelength converters are now dependent random variables. Therefore, the Bayes error is difficult to derive, we turn to study the upper bound based on the blocking probability $P_b$.

## C. Blocking Probability

To derive the blocking probability under the dependent model, we define $\alpha = \rho_2/\rho$, which characterizes the percentage of occupied wavelengths used for inter-domain calls. Then the independent model is just one special case of the dependent model with $\alpha = 0$ ($\rho_2 = 0$). From assumptions in Section *VI. A*, we have,

(*i*) $P$(wavelength $w_j$ is used for inter-domain call in subnet *i*| $w_j$ is not used for inter-domain call in subnet *i-1*)= $P_n$,

(*ii*) $P$(wavelength $w_j$ is used for inter-domain call in subnet *i*| $w_j$ is used for inter-domain in subnet *i-1*)= $P_n P_l + (1-P_l)$.

Therefore,

$$\rho_2 = (1-\rho_2)P_n + \rho_2[P_n P_l + (1-P_l)]. \qquad (11)$$

It follows that

$$P_n = \frac{\rho_2 P_l}{1-\rho_2(1-P_l)}. \qquad (12)$$

Define $I_i = 1$ if there is one wavelength continuous path within subnet *i*; and $I_i = 0$, otherwise. Then a decision that there are wavelengths available for an end-to-end call ($\omega = 1$) is equivalent to $I_i = 1$ for all *i*.

Let $M_i$ be the number of inter-domain connections in subnet *i*. Then the blocking probability under the dependent model can be expressed as:

$$P_{bd} = 1 - \sum_{M_1, M_2, ...M_L} \{P(I_1 = 1, I_2 = 1, ..., I_L = 1 | M_1, M_2, ..., M_L) P(M_1, M_2, ..., M_L)\}$$
$$= 1 - \sum_{M_1, M_2, ...M_L} \{P(I_1 = 1 | M_1) P(M_1) P(I_2 = 1 | M_2) P(M_2 | M_1) \cdots P(I_L = 1 | M_L) P(M_L | M_{L-1})\}, \qquad (13)$$

where

$$P(I_i = 1 | M_i) = 1 - [1-(1-\rho_c)^H]^{(F-M_i)}. \qquad (14)$$

Let $M_{1i}$ be the number of inter-domain calls in the *i*-th subnet that continue to the next subnet. We have

$$P(M_i = m_i | M_{i-1} = m_{i-1}) = \sum_{M_{1i-1}=0}^{\min\{m_i,\, m_{i-1}\}} P(M_i | M_{1i-1}) P(M_{1i-1} | M_{i-1}), \qquad (15)$$

where

$$P(M_{1i-1} = m | M_{i-1} = k) = \binom{k}{m} P_l^{(k-m)} (1-P_l)^m, \quad \text{for } 0 \le m \le k \le F, \qquad (16)$$

and

$$P(M_i = h | M_{1i-1} = m) = \binom{F-m}{h-m} P_n^{(h-m)}(1-P_n)^{(F-h)},$$
$$\text{for } 0 \le m \le h \le F. \quad (17)$$

Inserting Equations (14-17) into Equation (13), $P_{bd}$ can be computed efficiently using the forward part of the forward-backward algorithm [26].

### *D. Numerical Analysis*

The blocking probability does not have a close-form expression either, but can be evaluated numerically. Fig. 7 plots $P_{bd}$ vs. $\rho$ for $F = 120$, $H = 5$, $L = 3$, $\alpha = 0$, 0.6, 0.9, $P_l = 0.2$. It could be found that $\rho$ has a similar "threshold effect" on the value of $P_{bd}$ to that in the independent model. In addition, the threshold is increasing with $\alpha$, which is defined as the percentage of working wavelengths used for inter-domain calls. This, intuitively, is due to the fact that the dependence of wavelength usage introduced by inter-domain calls reduces the blocking probability for a given load $\rho$. When $\alpha = 0$, the dependent model is reduced to the independent model, and the threshold has the lowest value.

Fig. 8 plots $P_{bd}$ vs. $\rho$ for $F = 20$, 40, 120, $H = 5$, $L = 3$, $\alpha = 0.6$, $P_l = 0.2$. We can find that the threshold is increasing with the number of wavelengths *F*. This is due to the fact that the more wavelengths, the smaller the blocking probability for a given load. The sharpness of the transition also increases with respect to *F*, suggesting an asymptotic behavior of the blocking probability for a large *F*.

Fig. 9 plots the upper bound for the probability of error from Fig. 7 using Theorem 1. It shows that the value of $P_e$ is small under most load conditions.

### *E. Special Cases*

A question rises why the threshold effect persists for both independent and dependent models. We investigate this question by considering special cases when the number of wavelengths is large, and all the sub-networks are weakly-connected ($P_l$ is large). Under these conditions, analytical form of the blocking probability can be derived.

#### *1) Gaussian* Approximation

An important step to obtain a close form expression for the blocking probability is to approximate the joint probability of the local states ($N_i$'s) at wavelength converters. When the number of wavelengths *F* is large (and *L* is small), local states at wavelength converters, $X = (N_1, N_2, \cdots, N_L)$, are joint Gaussian random variables with probability $1 - O(\frac{L}{\sqrt{F}})$ [27]. Such a Guassian distribution can be completely characterized by the means, variances, and covariances of $N_i$'s. Specifically, all $N_i$'s are random variables with the same mean $\mu$ and variance $\sigma^2$, where

$$\mu = F\rho, \quad (18)$$

and

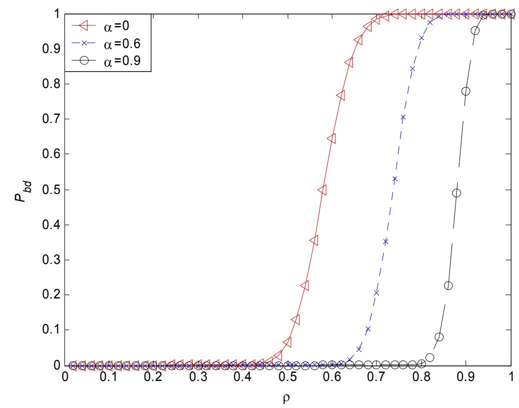


Fig. 7. Load ( $\rho$ ) vs. blocking probability ( $P_{bd}$ ): *F*=120, *H*=5, *L*=3, $\alpha = 0, 0.6, 0.9$, $P_l = 0.2$.

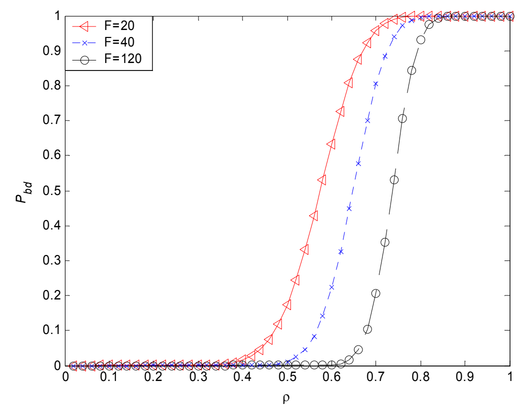


Fig. 8. Load ( $\rho$ ) vs. blocking Probability ( $P_{bd}$ ): *F*=20, 40, 120, *H*=5, *L*=3, $\alpha = 0.6$, $P_l = 0.2$.

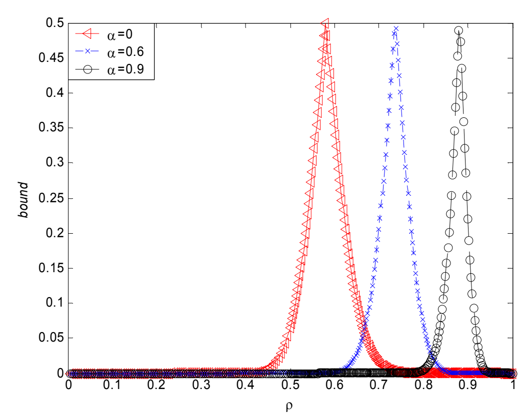


Fig. 9. Load ( $\rho$ ) vs. upper bound of $P_e$ : *F*=120, *H*=5, *L*=3, $\alpha = 0, 0.6, 0.9$, $P_l = 0.2$.

$$\sigma^2 = F\rho(1-\rho). \quad (19)$$

The covariance $C_{ij}$ between $N_i$ and $N_j$ for $i \ne j$ characterizes the dependence between two subnets, where

$$C_{ij} = E[N_i N_j] - \mu^2. \quad (20)$$

Such dependence can be further characterized through partitioning $N_i$ and $N_j$ into different components,

$$N_i = N_{ii} + M_i; \quad (21)$$

and

$$N_j = N_{jj} + M_j; \quad (22)$$

where

$N_{ii}$ is the number of wavelengths occupied by local calls at the first hop of the *i-th* subnet,

$N_{jj}$ is the number of wavelengths occupied by local calls at the first hop of the *j-th* subnet,

$M_i$ is the number of wavelengths in the *i-th* subnet occupied by inter-domain calls,

$M_j$ is the number of wavelengths in the *j-th* subnet occupied by inter-domain calls.

Define $\rho_g$ as the correlation coefficient between $N_i$ and $N_{i+1}$. Then $N_i$ and $N_{i+1}$ have a bivariate normal distribution: $P(N_i, N_{i+1}) \sim Normal(\mu, \mu, \sigma^2, \sigma^2, \rho_g)$. Since $N_1, N_2, \ldots, N_L$ form a Gaussian Markov Chain, the joint probability distribution of $X = (N_1, N_2, \cdots, N_L)$ is

$$P(N_1, N_2, \ldots, N_L) \sim Normal(\underline{\mu}, \Sigma), \quad (23)$$

where $P(N_1, N_2, \ldots, N_L) = \frac{1}{(2\pi)^{L/2} |\Sigma|^{1/2}} e^{-\frac{(X-\underline{\mu})\Sigma^{-1}(X-\underline{\mu})^T}{2}}$,

$$\underline{\mu} = \left[F\rho, F\rho, \cdots, F\rho\right]_{1\times L},$$

and

$$\Sigma^{-1} = \begin{bmatrix} a_{11} & a_{12} & 0 & \cdots & 0 & 0 \\ a_{12} & a_{22} & a_{23} & \cdots & 0 & 0 \\ 0 & a_{32} & a_{33} & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & a_{L-1\,L-1} & a_{L-1\,L} \\ 0 & 0 & 0 & \cdots & a_{L\,L-1} & a_{L\,L} \end{bmatrix}_{L\times L},$$

with

$$a_{ii} = \begin{cases} \frac{1}{(1-\rho_g{}^2)\sigma^2}, & for\ i = 1\ or\ i = L, \\ \frac{1+\rho_g{}^2}{(1-\rho_g{}^2)\sigma^2}, & otherwise, \end{cases}$$

and

$$a_{ij} = -\frac{\rho_g}{(1-\rho_g{}^2)\sigma^2}, \quad for\ i \neq j.$$

It can be shown that

$$\rho_g = \frac{(\rho_2 - P_n)(1-\rho)}{\rho(1-\rho_2)} = (\frac{1-\rho}{\rho})(\frac{\rho\alpha - P_n}{1-\rho\alpha}), \quad (24)$$

for all $i = 1, 2, \cdots, L$-1. (Detailed derivations can be found in *Appendix II*). Two observations can be made here:

($i$) $\rho_g$ is monotonically decreasing when $P_n$ is increasing. Specifically, when $\rho_2 = P_n$, all the inter-domain calls supported by a network domain exit at the current network domain ($\rho_g = 0$). When $P_n = 0$, all the inter-domain calls are end-to-end connections traversing all the network domains ($\rho_g = \frac{1-\rho}{1-\rho\alpha}\alpha$). Note that we always have $0 \le P_n \le \rho_2$.

($ii$) $\rho_g$ is monotonically increasing with respect to $\alpha$. where $\alpha$ is the percentage of inter-domain calls. For instance, when $\alpha = 0$, i.e., all the calls supported by the network are local calls, we have $\rho_g = 0$. When $\alpha = 1$, i.e., all the calls are inter-domain calls, we have $\rho_g = 1 - \frac{P_n}{\rho}$.

*2) Weakly*-Connected Sub-Networks

When $\rho_g = 0$, all sub-networks are completely decoupled, i.e., each inter-domain call lasts for one subnet ($P_l = 1$). The non-blocking probability of decoupled subnets, we have

$$P_{ad}^* = \left\{1 - [1-(1-\rho)(1-\rho_c)^{H-1}]^F\right\}^L, \quad (25)$$

where $\rho_c$ is the probability that a wavelength is used for local calls given that it is not used for inter-domain calls, $\rho_c = \rho_1 / (1-\rho_2)$. For the non-blocking probability of the independent model, we have in equation (6)

$$P_{ai} = \left\{1 - [1-(1-\rho)^H]^F\right\}^L.$$

Equation (25) bears a similar form to Equation (6), and thus it can be shown that there exists a threshold effect in the blocking probability for decoupled subnets similar to that for the independent model.

Of particular interest is when all the sub-networks are weakly connected. When $\rho_g$ is small, all sub-networks are weakly-connected, i.e., a small percentage of the calls are inter-domain calls ($\alpha$ is small), and/or inter-domain calls exit at current subnet with a large probability ($P_l$ is large). For weakly connected sub-networks, we obtain the following theorem through Taylor Expansion:

*Theorem 2. For weakly-connected sub-networks, i.e., $\alpha$ is small and/or $P_l$ is large, the non-blocking probability of the dependent model can be expressed as*

$$P_{ad} = P_{ad}^*(1+\eta) + o(\rho_g), \quad (26)$$

*where $P_{ad}^*$ is the non-blocking probability of the decoupled subnets as given in equation (25) , and $\eta$ is proportional to $\rho_g$ (see Appendix III for details).*

We can find that:

($i$) When $P_l = 1$, all inter-domain calls last one subnet. Hence all the sub-networks are decoupled, and we have $\eta = 0$, $P_{ad} = P_{ad}^*$.

($ii$) When $P_l$ is large (e.g., $P_l \ge 0.8$), a small percentage of the inter-domain calls last more than one subnets. Hence the sub-networks are weakly-connected, and $P_{ad} \approx P_{ad}^*(1+\eta)$. Here the non-blocking probability is just that of the decoupled sub-networks plus a small perturbation. Thus we can expect a threshold effect occurs under the weakly-connected sub-networks. The analysis here further explains why the threshold effect persists for both independent and dependent model.

## VII. Simulation Results

For more realistic scenarios with dynamic call arrivals and departures, the Bayesian approach we use would be applicable conceptually. However, the exact *a posterior* probability would be rather complex. Hence, a question is whether or not the static model we use could result in a good approximation. In this section, we investigate this issue through simulation of light-path assessment for dynamic call patterns. Of particular interest is the performance of the analytical bound on $P_e$, which is derived using the static model in a dynamic setting.

### A. Simulation Setup

We simulate light-path assessment in a network of bus topology with three network domains. Each network domain is assumed to have 5 hops. Connection requests are assumed to obey a Poisson Process with unit exponential holding time. Define $\lambda_l$ as the arrival rate of connection requests for local calls at each link, and $\lambda_{ij}$ as the arrival rate of connection requests for inter-domain calls from domain *i* to domain *j*. Note that the connections between two border nodes of domain *i* are considered as inter-domain calls from domain *i* to domain *i*. Let the total arrival rate to the network be $\lambda$, then $\lambda = 15\lambda_l + \sum_{i=1}^{1}\sum_{j=1}^{3}\lambda_{ij}$. Furthermore, following the assumptions in Section *VI*, we have

$$\alpha = \frac{\sum_{i=1}^{1}\sum_{j=1}^{3} 5(j-i+1)\lambda_{ij}}{15\lambda_l + \sum_{i=1}^{1}\sum_{j=1}^{3} 5(j-i+1)\lambda_{ij}}, \tag{27}$$

$$P_l = \frac{\sum_{i=1}^{m}\lambda_{im}}{\sum_{i=1}^{m}\sum_{j=m}^{3}\lambda_{ij}} \quad for\ m = 1, 2, \tag{28}$$

and

$$\sum_{i=1}^{m}\sum_{j=m}^{3}\lambda_{ij} \equiv const \overset{def}{=} \lambda_\Delta. \ for\ m = 1,2,3. \tag{29}$$

$\lambda_\Delta$ can be considered as the total arrival rate for inter-domain connections at each network domain. Solving equations (27-29), we can obtain the arrival rates for connection requests with different sources and destinations as follows:

$$\begin{aligned} &\lambda_l = \frac{(1-\alpha)}{\alpha}\lambda_\Delta; \\ &\lambda_{11} = \lambda_\Delta P_l;\ \lambda_{12} = \lambda_\Delta P_l(1-P_l);\ \lambda_{13} = \lambda_\Delta(1-P_l)^2; \\ &\lambda_{22} = \lambda_\Delta P_l^2;\ \lambda_{23} = \lambda_\Delta P_l(1-P_l); \\ &\lambda_{33} = \lambda_\Delta P_l, \end{aligned} \tag{30}$$

where $\lambda_\Delta = \dfrac{\lambda\alpha}{15-(14-2P_l)\alpha}$.

For a specific network load $\rho$, we adjust the total traffic arrival rate to the network $\lambda$ to be either high or low, so that the probability that a wavelength is used in the network remains approximately $\rho$.

The simulator is based on discrete event simulation. For each simulation, 10 runs are performed where each run consists of 100,000 end-to-end connection requests. Four decisions may result from light-path assessment using the partial information discussed previously: (i) correct acceptation (*CA*), (ii) incorrect acceptation (*IA*), (iii) correct rejection (*CR*), (iv) incorrect rejection (*IR*). The probability of error is obtained as the percentage of *IA* and *IR* out of all the decisions.

### B. Simulation Results

Fig. 10 depicts the probability of error for light-path assessment using aggregated information with $F = 40$, $\alpha = 0.6$, and $P_l = 0.2$. The reason for choosing these parameters are: (1) more wavelengths would be used for inter-domain calls than for local calls; (2) a large percentage of inter-domain calls supported by one network domain would be calls passing through that domain.

The simulation result confirms the threshold effect that is predicted by the analytical model and shows the good performance of the analytical bound. We can find that, using Bayesian approach based on only aggregated information and the static model, $P_e$ is neglible under most load conditions; and $P_e$ increases to its peak exponentially when the load is close to the threshold. Furthermore, the static model predicts the threshold of the load accurately. For Fig. 10, $\rho_{threshold} \approx 0.65$.

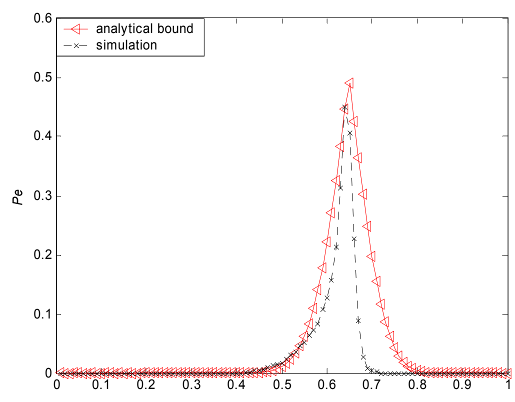


Fig. 10. Analytical bound and simulated $P_e$ : *F=40*, *H=5*, *L=3*, $\alpha = 0.6$, $P_l = 0.2$.

Fig. 11 shows the simulation results for $F = 80$, $\alpha = 0.6$, and $P_l = 0.2$. It also confirms that the Bayesian approach could give us a small $P_e$ except when the load is in a small region close to the threshold. When the load is close to the threshold ($\rho_{threshold} \approx 0.71$ for Fig. 11), $P_e$ increases to its peak exponentially. Because of its static nature, the dependent model used in simulation cannot capture the instantaneous blocking probability of the network carrying dynamic traffic with 100% accuracy. Therefore, the probability of error exceeds 0.5 when the load is at the threshold. However, simulation results confirm that it is possible to achieve a probability of error close to 0 using only aggregated information and a static model of blocking probability.

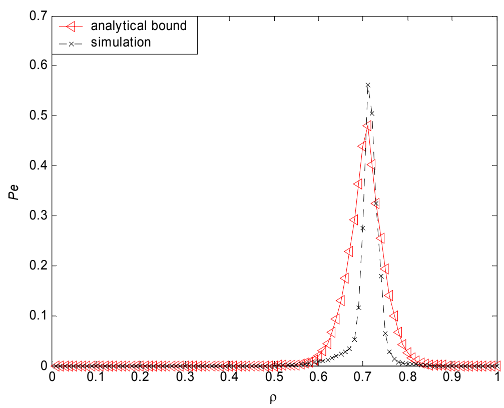


Fig. 11. Analytical bound and simulated $P_e$ : *F=80*, *H=5*, *L=3*, $\alpha = 0.6$, $P_l = 0.2$.

## VIII. CONCLUSION

In this work, we have investigated network management information for light-path assessment across administrative domains (subnets). Our focus has been on studying the scalability of management information, which includes aggregated information of each subnet, and local information from wavelength converters on network boundaries.

We have formulated the problem based on decision theory, and defined the performance of using partial management information through the Bayes probability of error. A bound in terms of blocking probability is derived to estimate such a performance. We then defined the scalability of management information as the growth rate with respect to network size and resource when a desired performance is achieved.

A scalable case has been studied where the partial management information grows only logarithmically with the number of wavelengths per link. Our study reveals that when the number of wavelengths is large, the resulting Bayes error is negligibly small for most of the network load conditions. Therefore, a small loss in performance (the Bayes error) may be traded off with a large saving in network management information. In other words, the abundant network resource, which is the large number of wavelengths in future WDM networks, may make it possible to reduce the amount of network management information while achieving a good performance.

The problem of light-path assessment is related to wavelength routing. One thought resulting from this work is to use aggregated information for wavelength routing when it is impractical to flood detailed link state information across the whole network. For instance, light-path assessment could be done for each candidate route based on aggregated information from each network domain and instantaneous measurements from a limited number of links. The optimal route can then be chosen accordingly. Detailed relationships need to be derived between light-path assessment and wavelength routing, which can be one of the extensions to this work

## APPENDIX I

### PROOF OF THEOREM I

*Proof:*

Consider the following *a posterior* probability:

$$f(x) = P(\omega = 1 | X = x).$$

The Bayes rule decides

$$\begin{cases} \omega = 1 & if \quad f(x) \ge 1/2, \\ \omega = 0 & otherwise, \end{cases}$$

Therefore,

$$P_e = \sum_X P_{e|X=x} P(X = x)$$

$$= \sum_X P(X = x) \min\{f(x), (1 - f(x))\}$$

$$\le \min\{\sum_X P(X = x) f(x), \sum_X P(X = x)(1 - f(x))\}.$$

Since $P_b = 1 - \sum_X P(X = x) f(x)$, we have

$$0 \le P_e \le \min\{P_b, (1 - P_b)\}.$$

□

## APPENDIX II

### DERIVATION OF THE CORRELATION COEFFICIENT $\rho_g$

Let $W_{mi} = 1$ if wavelength *m* is used for an inter-domain call at domain *i*, $W_{mi} = 0$ otherwise; Let $L_{mi} = 1$ if wavelength *m* is used for a local call at the first link of domain *i*, $L_{mi} = 0$ otherwise, where $m = 1, 2, \cdots, F$, $i = 1, 2, \cdots, L$. From the assumptions in *Section VI. A.*, the following joint probabilities hold for $i = 1, 2, \cdots, L - 1$:

$$P(W_{mi} = 1, W_{m\,i+1} = 1) = \rho_2 [P_n P_l + (1 - P_l)]; \tag{31}$$

$$\begin{aligned} P(W_{mi} = 1, L_{m\,i+1} = 1) &= \sum_{k=0}^{1} P(W_{mi} = 1, W_{m\,i+1} = k, L_{m\,i+1} = 1) \\ &= \sum_{k=0}^{1} P(W_{mi} = 1) P(W_{m\,i+1} = k | W_{mi} = 1) \\ &\quad \cdot P(L_{m\,i+1} = 1 | W_{mi} = 1, W_{m\,i+1} = k) \\ &= P(W_{mi} = 1) P(W_{m\,i+1} = 0 | W_{mi} = 1) \\ &\quad \cdot P(L_{m\,i+1} = 1 | W_{mi} = 1, W_{m\,i+1} = 0) \\ &= \rho_2 (P_l - P_n P_l)(\frac{\rho_1}{1 - \rho_2}); \end{aligned} \tag{32}$$

$$P(L_{mi}=1, L_{m\,i+1}=1)=\sum_{k=0}^{1}P(L_{mi}=1, W_{m\,i+1}=k, L_{m\,i+1}=1)$$
$$=\sum_{k=0}^{1}P(L_{mi}=1)P(W_{m\,i+1}=k \mid L_{mi}=1)$$
$$\cdot P(L_{m\,i+1}=1 \mid L_{mi}=1, W_{m\,i+1}=k)$$
$$=P(L_{mi}=1)P(W_{m\,i+1}=0 \mid L_{mi}=1)$$
$$\cdot P(L_{m\,i+1}=1 \mid L_{mi}=1, W_{m\,i+1}=0) \quad (33)$$
$$=\rho_1(1-P_n)(\frac{\rho_1}{1-\rho_2});$$
$$P(L_{mi}=1, W_{m\,i+1}=1)=\rho_1 P_n. \quad (34)$$

Then,

$$C_{i\,i+1}=E[N_i N_{i+1}]-F^2\rho^2$$
$$=E[N_{ii}N_{i+1\,i+1}]+E[N_{ii}M_{i+1}]+E[M_i N_{i+1\,i+1}] \quad (35)$$
$$+E[M_i M_{i+1}]-F^2\rho^2.$$

From Equation (31), we have,

$$E[N_{ii}N_{i+1\,i+1}]$$
$$=E[\sum_{m=1}^{F}L_{mi}\sum_{n=1}^{F}L_{n\,i+1}]$$
$$=E[\sum_{m=1}^{F}L_{mi}L_{m\,i+1}]+F(F-1)\rho_1^2$$
$$=FP(L_{mi}=1, L_{m\,i+1}=1)+F(F-1)\rho_1^2$$
$$=F\rho_1(1-P_n)(\frac{\rho_1}{1-\rho_2})+F(F-1)\rho_1^2.$$

Similarly, we have

$$E[N_{ii}M_{i+1}]=E[\sum_{m=1}^{F}L_{mi}\sum_{n=1}^{F}W_{n\,i+1}]$$
$$=F\rho_1 P_n+F(F-1)\rho_1\rho_2;$$
$$E[M_i N_{i+1\,i+1}]=E[\sum_{m=1}^{F}W_{mi}\sum_{n=1}^{F}L_{n\,i+1}]$$
$$=F\rho_2(P_l-P_nP_l)(\frac{\rho_1}{1-\rho_2})+F(F-1)\rho_1\rho_2;$$
$$E[M_i M_{i+1}]=E[\sum_{m=1}^{F}W_{mi}\sum_{n=1}^{F}W_{n\,i+1}]$$
$$=F\rho_2(P_nP_l+1-P_l)+F(F-1)\rho_2^2.$$

From Equation 11, we have

$$\rho_2=(1-\rho_2)P_n+\rho_2[P_nP_l+(1-P_l)].$$

Then $C_{i\,i+1}$ can be simplified as

$$C_{i\,i+1}=F[\frac{(1-\rho)^2(\rho_2-P_n)}{(1-\rho_2)}]. \quad (36)$$

Therefore,

$$\rho_g=\frac{C_{ij}}{\sigma^2}=\frac{C_{ij}}{F\rho(1-\rho)}=\frac{(1-\rho)(\rho_2-P_n)}{(1-\rho_2)\rho}.$$

□

## APPENDIX III
### PROOF OF THEOREM II

*Proof*:

The non-blocking probability of the dependent model satisfies $P_{ad}=E[f(\omega=1 \mid X)]$. Since $X=(N_1,...,N_L)$ are jointly Gaussian, we can expand $P_{ad}$ in terms of $\rho_{ij}$'s as follows,

$$P_{ad}=P_{ad}^*+\frac{\partial P_{ad}}{\partial\rho_g}\Big|_{\rho_g=0}\rho_g+o(\rho_g), \quad (37)$$

where $P_{ad}^*$ is the non-blocking probability of the dependent model when all the inter-domain calls only last for one subnet ($P_l=1$). Specifically, we have

$$P_{ad}^*=\left\{1-[1-(1-\rho)(1-\rho_c)^{H-1}]^F\right\}^L. \quad (38)$$

Let $\gamma=[1-(1-\rho_c)^{H-1}]$, then

$$\frac{\partial P_{ad}}{\partial\rho_g}$$
$$=\frac{\partial}{\partial\rho_g}(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}P(N_1,N_2,\cdots,N_L)\prod_{i=1}^{L}(1-\gamma^{(F-N_i)})dN_1dN_2\ldots dN_L)$$
$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\frac{\partial}{\partial\rho_g}(P(N_1,N_2,\cdots,N_L))\prod_{i=1}^{L}(1-\gamma^{(F-N_i)})dN_1dN_2\ldots dN_L.$$

When $\rho_g=0$,

$$\frac{\partial P_{ad}}{\partial\rho_g}=\left\{1-[1-(1-\rho)(1-\rho_c)^{H-1}]^F\right\}^{L-2}q_{ij},$$

where

$$q_{ij}=(L-1)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(1-\gamma^{F-N_i})(1-\gamma^{F-N_j})\frac{\partial f(\pi_i,\pi_j;\rho_{ij})}{\partial\rho_{ij}}\Big|_{\rho_{i,j=0}}dN_idN_j,$$

with $f(\pi_i,\pi_j;\rho_{ij})$ being the joint Gaussian p.d.f. of $N_i$ and $N_j$. Simplifying $q_{ij}$, we have,

$$q_{ij}=\frac{(L-1)}{F\rho(1-\rho)}\{\int\int_{-\infty}^{\infty}f(\pi_i,\pi_j,0)(N_i-F\rho)(N_j-F\rho)$$
$$(1-\gamma^{F-N_i})(1-\gamma^{F-N_j})dN_idN_j\}.$$

Using the characteristic functions of Gaussian r.v.'s, we have

$$q_{ij}=(L-1)\gamma^{2(F-\mu)+\sigma^2\ln\gamma}\sigma^2\ln^2\gamma, \quad (39)$$

Therefore, we have

$$P_{ad}=P_{ad}^*(1+\eta)+o(\rho_g),$$

where

$$\eta=\frac{q_{ij}}{\{1-[1-(1-\rho)(1-\rho_c)^{H-1}]^F\}^2}\rho_g. \quad (40)$$

□

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